Scaling As A Fundamental Part Of Physic

Drop an ant and an elephant from the Eiffel Tower. The ant lands softly and survives. The elephant goes splat and ruins the day of any nearby tourists.

An ant can lift a multiple of its own weight. An elephant can lift only a fraction of its own weight.

A small TV costs $100. A TV twice as wide costs $400.

What gives? Why is the universe like this?

The answer to this puzzle is called scaling and it is a fundamental part of physics. Learn this and you will understand many things about this world.

Linear Relationships

The equation y = x is known as a linear equation. Whatever the term 'y' represents is dependent directly on whatever the term 'x' represents. 'X' could be your paycheck and 'y' could be your bank account. Double your salary and your bank account will go up twice as fast (if you are disciplined and stay away from Amazon).

A better example is if you are walking at 5 kilometres per hour. In one hour you go 5 kilometres. In two hours you go 10 kilometres. A very simple and direct relationship.

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Parabolic Relationships

The equation y = x2 is known as a parabolic equation. In this case 'y' depends on 'x' squared.

A good example is the area of a square. Take a square that is x units wide. The area of that square is x times x equals x2.

Take another square that is twice as wide (2x units wide). The area of that square is 2x times 2x equals 4x2.

If x =1 then the small square will have an area of 1 x 1 = 1 and the large square will have an area of 2 x 2 = 4.

Making TVs

This is why the price of TV's that are twice as large tend to cost something like 4 times as much. In the larger TV there are 4 times as many pixels that the factory has to produce without flaw and the odds of too many of them being dead pixels increases with the area of the panel (there is more to making a TV than that but this gives you the general idea).

More material plus a greater probability of dead pixels means cost increases faster than a simple linear relationship.

Verification

I checked this conclusion on Amazon for a range of VIZIO brand TVs. The relationship holds very well when comparing the 50 inch TV to the 25 inch TV. The relationship holds less well when comparing the 32 inch TV to the 25 inch TV because you still need to produce the supporting circuit boards and power supplies which tend to scale in price differently.

The Strength of Bones

Another good example is the strength of your bones. The strength of bones increases roughly as the area of the bone which is related the square of the width of its cross-section. Make a bone twice as wide and it can withstand four times the force before breaking.

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Cubic Relationships

The equation y = x3 is known as a cubic equation. Whatever the term 'y' represents is whatever the term 'x' represents cubed.

A good example is the volume of a cube. Take a cube that is x units wide. The volume of that square is x times x times x equals x3.

Take another cube that is twice as wide or 2x units wide. The area of that square is 2x times 2x times 2x equals 8x3.

If x = 1 then the small cube will have an area of 1 x1 x 1 = 1 and the large square will have an area of 2 x 2 x 2 = 8.

A better example is the volume of a sphere which is V = 4 ? r3. This is a better example because later in the post we will approximate our ant and our elephant as a spheres.

Why approximate an elephant as a sphere? Because we are physicists that's why.

Y FOR U GO SPLAT?

Okay, so now we drop our ant and our elephant from the Eiffel Tower. There is a lot going on here in the trip to the ground so let's walk through each thing one at a time.

Air Resistance

The air resistance will scale roughly as the area of an object which is the square of its length.

Mass

The mass of an object scales roughly as the volume of an object. Let's assume both the ant and the elephant are spheres. Let's say the elephant is 1000 times longer than the ant. It will therefore have a mass 1000 cubed equals 1 billion more than the ant.

Checking some numbers on the internet gives a mass of an ant at between 1 and 5 milligrams and the mass of an elephant at 3000 kg. The ratio comes out to between roughly 600 million and 3 billion. Not bad for a wild guess.

Force of Gravity

The force pulling down on the elephant will be $F = mg$ and since mass is the length of the object cubed the mass will be the length of the object cubed. The force pulling down on the elephant will be much larger.

Total Acceleration

The force upwards, air resistance, goes up as the square of an object's size but the force downwards goes up as the cube of an object size. The two will tend to cancel each other out leaving a very rough linear relationship for the actual downward acceleration. The larger you are the faster you accelerate in free fall (in air, not in vacuum).

This means that the acceleration of the elephant will be much faster than the acceleration of the ant.

The ant will tend to waft down whereas the elephant will plummet.

It's not looking good for Mr. Elephant.

Bone and Body Strength

The strength of the bones and other structures within the bodies of our two creatures will tend increase with the square of the size of the dimension. These bones will have to support the mass of the creature which goes up as the cube of the creature's dimensions.

So it is harder for bones and other structures to support the increase in mass as the size of an animal increases.

Image credit: Bill Haywood link

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This is why there is an upper size limit for land creatures. The situation in water is somewhat different so we can get larger creatures like the Blue Whale.

THE END RESULT

So because mass tends to scale as the cube of a creature's size, strength tends to scale as the square of a creature's size and the rate of falling in air is faster for larger objects the impact at the end of the trip is a very different experience for an ant and an elephant.

This is why scientists need to wear sealed hazmat suits when dropping elephants from tall towers (gross).