The Space-Time CE/SE Method For Solving Two-Phase And Two- Dimensional Shallow Flow Model

Here we will be presenting the space-time conservation element and solution element (CE/SE) technique to compute the numerical solution of two-phase and two-dimensional shallow water equations along with bottom topography. Flow equations consist of continuity and momentum equations. The model is hyperbolic and contains the non-conservative terms in the momentum equation increasing the complexity of the model. Shallow water flow equations have numerous applications in the fields of environmental as well as hydraulic engineering.

Moreover, these flow equations are encountered in estuary, coastal regions of tidal flows, dams, reservoirs, coastal areas and in flow of rivers, near-shore regions and bays. The exact solution of the above said model is not available in the literature. A finite volume based CE/SE scheme will be applied to solve considered model equations. The numerical computation of shallow two-phase water flow equations is the challenging problem in computational fluid dynamics. That’s why we used the CE/SE technique to solve this system. The CE/SE scheme was initially spread by S. C. Chang for a time-accurate, solution of hyperbolic conservation laws. To determine the potential and accuracy of the present technique, a number of case studies will be presented.

To validate, the simulations of the CE/SE technique will be equated with a high resolution kinetic flux-vector splitting scheme (KFVS). IThe shallow two-phase water equations are mainly used to relate shallow layer of the combination of solid granular material and a fluid that are near about straight. These equations are basically derived from the Euler equations through perpendicularly averaging of the phase depth. The phases are characterized by difference of their densities. The model equations comprises of source terms which appear because of bottom topography and non-conservative products responsible for momentum exchange between the phases.

The numerical solution of these equations having practical demand, whose big domains are lie in space and time, having calculation is very costly. Shallow water flows can be characterized as various forms of flows not only involving water as fluid. In such flows where the straight lengths is much larger than the perpendicular length and this is true for several physical processes. Such models comprises of system of equations consisting of mass and momentum laws for the solid and fluid phases, having both non-conservative and conservative momentum terms. The studied system of shallow two-phase water equations are the extensions of Saint-Venant system proposed by A. J. C. de-Saint-Venant in [2]. Several numerical schemes in recent years have been developed to solve the system of shallow two-phase water equations under consideration, for example, Central Scheme by J. Balbas in [7], Central Upwind Scheme by A. Kurganov in, fast finite volume solver by E. Audusses for hydrostatic shallow multi-layered water flows in [11], finite volume method by C. Berthon in [12], Discontinuous Galerkin method proposed by N. Izem and M. Wakrim in [13]. Although, these methods has high rate of accuracy, but the methods have one major drawback that they may produce negative unphysical values of the flow height. Also, central scheme for hyperbolic conservations laws by A. Kurganov [14-15]. Importantly, S. Qamar et. al [17] applied CE/SE to solve, single and two-phase one-dimensional shallow equations.